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On the Motion of a Particle with Anisotropic Mass

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Abstract

On the basis of the hypothesis that particle mass is anisotropic rather than isotropic, we investigate the relativistic motion of a particle within the framework of flat space-time. Assuming that the mass anisotropy is associated with the photon cloud of the particle, we argue that the self-energy of a particle is of the order of magnitude $|\delta m/M_0| \sim 0{\cdot}0005$, for which conventional quantum electrodynamics, however, gives an infinite value.

1. *Introduction*

According to Mach's principle 'the inertial mass of a body is determined by the total distribution of matter in the universe', the value of inertial mass of a body depends on its direction of acceleration if the matter distribution is not isotropic. Therefore, mass is a tensor rather than a scalar quantity.

Cocconi & Salpeter (1958, 1960) suggested that if Mach's principle was true, the effects of this tensor inertial-mass would appear as a spatial anisotropy in certain experiments. It is, however, shown by several experiments (Drever, 1961; Hughs *et al.*, 1960) that the anisotropy of inertial mass is extremely small in a reference frame connected with the earth. The ratio of the anisotropic part of the proton inertial mass to the isotropic part is of the order of 5×10^{-23}.

Consequently, the isotropy of the mass tensor appears to be an experimental fact. We wish to emphasise, however, that negative results obtained by the experiments of such kinds do not rule out all the possibilities of different kinds of mass anisotropy.

Sazonov (1972) has discussed the anisotropy of relativistic mass associated with the anomaly of time component in the mass tensor. However, Sazonov's theory violates the principle of covariance which is a fundamental requirement for the relativistic theory.

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From the point of view of quantum electrodynamics, the particle is always accompanied by a photon cloud and its bare mass is not observable. The particle mass defined in classical mechanics as an observable quantity is apparently considered to be the bare mass, because the concept of photon cloud is not introduced into classical mechanics. It is accepted that the special theory of relativity is valid for macroscopic as well as microscopic motion of the particle. Therefore, the photon cloud which is a microscopic entity may also be taken into account to describe the relativistic motion of a particle, and then one might be able to expect the possible appearance of new physical laws. In this paper, we shall attempt to construct a relativistic theory of a particle with anisotropic mass associated with the photon cloud.

2. *Theory*

2.1. *The Special Theory of Relativity for the Particle with Isotropic Mass*

Let us, first, briefly consider the ordinary theory of special relativity so as to make the discussion of the motion of a particle with anisotropic mass very clear.

Particle mass is generally defined by a scalar coefficient in the following expression for the action of the particle,

$$L = -m_0 c \int ds \tag{2.1.1}$$

where c is the velocity of light. The line element is usually given as

$$ds = \sqrt{(g_{ij}\, dx_i\, dx_j)} \tag{2.1.2}$$

with the metric tensor $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$ and $dx_0 = cdt$. The four momentum of the particle is given by

$$p_i = -g_{ij} \frac{\delta L}{\delta x_j} \equiv -g_{ij} \frac{\partial (dL)}{\partial (dx_j)} \tag{2.1.3}$$

Performing this differentiation, we obtain the well known formulae,

$$p_0 = \frac{m_0 c}{\sqrt{(1-\beta^2)}} \tag{2.1.4}$$

$$p_\alpha = \frac{m_0 v_\alpha}{\sqrt{(1-\beta^2)}} \qquad (\alpha = 1, 2, 3) \tag{2.1.5}$$

where $\beta = v/c$. From these relations, (2.1.4) and (2.1.5), the particle dispersion law yields

$$p_0^2 - \mathbf{p}^2 = m_0^2 c^2 \tag{2.1.6}$$

From the definition of energy $E = cp_0$, we can also obtain

$$E(v) = \frac{m_0 c^2}{\sqrt{(1-\beta^2)}} \tag{2.1.7}$$

for the velocity dependence of energy and

$$E(p) = \sqrt{(m_0{}^2c^4 + \mathbf{p}^2c^2)} \tag{2.1.8}$$

for the momentum dependence of energy. The ordinary Lorentz transformation under which the line element (2.1.2) and consequently the action of the particle (2.1.1) are invariant is

$$x_i' = \Lambda_{ij}x_j \tag{2.1.9}$$

where

$$\Lambda = \begin{bmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \qquad (\gamma = (1-\beta^2)^{-1/2}) \tag{2.1.10}$$

The transformation for four momenta can be written as

$$p_i' = \Lambda_{ij}^T p_j \tag{2.1.11}$$

where Λ^T is the transpose of the matrix Λ. The fact that the dispersion law (2.1.6) is invariant under transformation can easily be proved as follows:

$$p'^Tgp' = (\Lambda^Tp)^Tg(\Lambda^Tp) = p^T(\Lambda g\Lambda^T)p = p^Tgp$$

where we used the relation $\Lambda g\Lambda^T = g$.

2.2. *General Form of the Action of the Particle*

If the mass of the particle is assumed to be a tensor m_{ij}, it becomes possible to rewrite the action of the particle in the general form

$$L = -c\int\sqrt{(g_{ij}m_{ik}m_{jl}\,dx_k\,dx_l)} \tag{2.2.1}$$

When the mass tensor is written as

$$m_{ik} = M_0S_{ik} \tag{2.2.2}$$

where M_0 is referred to as isotropic mass, the action of the particle (2.2.1) leads to

$$L = -M_0c\int\sqrt{(U_{kl}\,dx_k\,dx_l)} \tag{2.2.3}$$

where we have introduced a new tensor

$$U_{kl} = g_{ij}S_{ik}S_{jl} \tag{2.2.4}$$

Then, the line element of our space-time is

$$(ds)^2 = U_{kl}\,dx_k\,dx_l \tag{2.2.5}$$

which is, generally, not invariant under the ordinary Lorentz transformation (2.1.9) for the case of $U_{kl} \neq g_{kl}$. We must find out a new Lorentz transformation under which the line element (2.2.5) and consequently the action of the particle (2.2.3) are invariant. In view of the experimental results (Drever, 1961; Hughes *et al.*, 1960) for inertial mass of the particle, the tensor U_{kl} is assumed to be spatially isotropic, i.e.

$$U = \begin{bmatrix} \frac{1}{G_0} & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \tag{2.2.6}$$

where G_0 is a constant on which all characters of the mass anisotropy are imposed. The constant b defined through the relation $G_0 = (1 + b)^2$ will be referred to as the anisotropy constant. It is clear that the ordinary theory of special relativity corresponds to a special choice of the metric tensor $U_{00} = -U_{11} = -U_{22} = -U_{33} = 1$.

2.3. *Relativistic Theory of the Motion of the Particle with Anisotropic Mass*

In this section, we shall discuss the case of the particle with anisotropic mass. A relation will be derived between the isotropic and anisotropic masses.

If we introduce a new constant

$$c_0 = c/\sqrt{G_0} \tag{2.3.1}$$

it is possible to write down the line element (2.2.5) in the form

$$ds = \sqrt{(U_{ij}\, dx_i\, dx_j)} = \sqrt{(g_{ij}\, dX_i\, dX_j)} \tag{2.3.2}$$

where

$$\begin{aligned} dX_0 &= \frac{1}{\sqrt{G_0}}\, dx_0 = c_0\, dt \\ dX_\alpha &= dx_\alpha \end{aligned} \tag{2.3.3}$$

The action of the particle, then, leads to

$$\begin{aligned} L &= -M_0 c \int \sqrt{(g_{ij}\, dX_i\, dX_j)} \\ &= -m_* c_0 \int \sqrt{(g_{ij}\, dX_i\, dX_j)} \end{aligned} \tag{2.3.4}$$

where

$$m_* = \sqrt{(G_0)} M_0 \tag{2.3.5}$$

and the corresponding four momentum is given by

$$P_i = -g_{ij} \frac{\delta L}{\delta X_j} = -g_{ij} \frac{\partial(dL)}{\partial(dX_j)} \tag{2.3.6}$$

From equation (2.3.6), we find, on performing the differentiation,

$$P_0 = \frac{m_* c_0}{\sqrt{(1-\beta_0^2)}} \tag{2.3.7}$$

$$P_\alpha = \frac{m_* v_\alpha}{\sqrt{(1-\beta_0^2)}} \qquad (\alpha = 1, 2, 3) \tag{2.3.8}$$

where

$$\beta_0 = v/c_0 \tag{2.3.9}$$

and consequently the dispersion law leads to

$$P_0^2 - \mathbf{P}^2 = m_*^2 c_0^2 \tag{2.3.10}$$

These relations, (2.3.7), (2.3.8) and (2.3.10), with the constant c_0 corresponding to the velocity of light are identical in the mathematical forms with those given in (2.1.4), (2.1.5) and (2.1.6), respectively. Therefore, the energy is given as

$$E(v) = c_0 P_0 = \frac{m_* c_0^2}{\sqrt{(1-\beta_0^2)}} \tag{2.3.11}$$

or

$$E(P) = \sqrt{(m_*^2 c_0^4 + \mathbf{P}^2 c_0^2)} \tag{2.3.12}$$

Accordingly, the Lorentz transformation matrix for X_i and P_i is also expressed by (2.1.11) provided that the velocity of light c is replaced by c_0.

The particle velocity is given by differentiating the energy (2.3.12) with respect to the momentum P_α,

$$v_\alpha = \frac{\partial E(P)}{\partial P_\alpha} = \frac{c_0^2 P_\alpha}{E(P)} = \frac{c_0 P_\alpha}{P_0} \tag{2.3.13}$$

which is, of course, consistent with the formulae obtained by the alternative way, i.e.

$$v_\alpha = \frac{dx_\alpha}{dt} = c_0 \frac{dX_\alpha}{dX_0} = c_0 \left[\frac{P_\alpha}{m_* c_0}\sqrt{(g_{ij}\, dX_i\, dX_j)}\right] \Bigg/ \left[\frac{P_0}{m_* c_0}\sqrt{(g_{ij}\, dX_i\, dX_j)}\right]$$

$$= \frac{c_0 P_\alpha}{P_0} \tag{2.3.14}$$

In getting the above relation we have used the fact that

$$P_i = -g_{ij}\frac{\delta L}{\delta X_j} = \frac{m_* c_0\, dX_i}{\sqrt{(g_{ij}\, dX_i\, dX_j)}} \tag{2.3.15}$$

When the isotropic transverse mass of the particle is written as

$$M = M_0/\sqrt{(1-\beta_0^2)} \tag{2.3.16}$$

the expression for the energy of the particle (2.3.11) leads to

$$E = \sqrt{(G_0)}Mc_0^2 \equiv \tilde{m}_* c_0^2 \tag{2.3.17}$$

We thus see that all the formulae for the anisotropic mass case can be obtained from the ordinary case by replacing the constant c and isotropic mass by the constant c_0 and anisotropic mass, respectively. Consequently, it is evident that the principle of covariance holds for the case under consideration. It is, thus, natural to interpret c_0 as the velocity of light for the mass anisotropic theory, in which the anisotropic mass m_* is the experimental mass. The equation (2.3.5) indicates the relation between the bare mass M_0 and the experimental mass m_*, although, so far, the constant G_0 remains undetermined. We shall give an estimate of G_0 later on.

It is important to notice that the mass appearing in the expressions of the energy, (2.3.11) and (2.3.12), is not the bare mass M_0 but the anisotropic mass M_*. On the other hand, in the ordinary theory, all the masses appearing in the corresponding formulae are isotropic masses, i.e. bare masses. Since the expression for energy has the same mathematical forms for both anisotropic and isotropic mass theories, the ordinary theory gives always correct answers as far as bare mass is replaced by experimental one. It should be pointed out, however, that our energy expression, (2.3.11) and (2.3.12), can not be directly reduced from the ordinary ones, (2.1.7) and (2.1.8), by using the relations (2.3.1) and (2.3.5).

According to quantum electrodynamics, the particle is always accompanied by its self-energy which is not taken into account in the ordinary theory of special relativity and that the bare mass is not observable. In the present theory, the mass correction associated with self-energy is considered as mass anisotropy and bare mass is clearly distinguished from experimental mass.

Finally, we remark that the transformation

$$x_i' = \tilde{\Lambda}_{ij} x_j$$

where

$$\tilde{\Lambda} = \begin{bmatrix} \gamma_0 & -\sqrt{(G_0)}\beta_0\gamma_0 & & \\ -\beta_0\gamma_0/\sqrt{G_0} & \gamma_0 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \qquad (\gamma_0 = 1/\sqrt{(1-\beta_0^2)})$$

makes the line element (2.2.5) invariant. However, it is no longer necessary because our space-time shifts from $U_{ij}\,dx_i\,dx_j$ to $g_{ij}\,dX_i\,dX_j$.

3. *Estimate of the Anisotropy Constant b*

As was shown in the previous section, the physical laws for the case under consideration have the same mathematical forms as those for the ordinary case except replacing isotropic mass by anisotropic one. However, the difference between mass isotropic and mass anisotropic theories appears in physical concept of space-time. In this section, we try to estimate the magnitude of anisotropic constant b. According to Sarantsev (1966), electron accelerators of energy up to 6 GeV which are in operation at present time confirm that the formulae derived from Einstein's theory of relativity, i.e.

$$m = \frac{m_0}{\sqrt{(1-\beta^2)}} \tag{3.1}$$

holds with an accuracy of $|\Delta m/m| \sim 5 \times 10^{-4}$. It turns out that for $|\Delta m/m| \sim 5 \times 10^{-4}$ a complete dephasing occurred and consequently the acceleration process completely breaks down. Making use of this experimental result, we put an upper limit on the anisotropy constant b in the following way.

The particle acceleration is generally performed by the transfer of the energy produced by the accelerator to the particle. The particle velocity is determined by the relativistic formula

(Total energy) = (kinetic energy) + (rest mass energy)

where the kinetic energy is just the energy which the machine transfers to the particle. The total energy is given by the rest mass energy multiplied by the Lorentz factor. Therefore, the kinetic energy transferred by the accelerator to the particle is independent of the relativistic formulae. In the present theory for the particle with anisotropic mass m_*, the energy can be written as

$$\tilde{m}_* c_0^2 = T + m_* c_0^2 \tag{3.2}$$

where c_0 is the velocity of light which is measured by experiments. For the conventional theory, i.e. the mass isotropic theory, we have

$$mc^2 = T + m_0 c^2 \tag{3.3}$$

where m_0 is the isotropic mass and c is the measured value for the velocity of light. When equation (3.2) is rewritten as

$$Mc_0^2 = \frac{T}{\sqrt{G_0}} + M_0 c_0^2 \tag{3.4}$$

M_0 and c_0 indicate the same quantities as m_0 and c, respectively. Then, from equations (3.3) and (3.4), we get the relation

$$\frac{\Delta m}{m} = \frac{m-M}{m} = 1 - \frac{T/\sqrt{(G_0)} + m_0 c^2}{T + m_0 c^2} \tag{3.5}$$

After a trivial algebra, we find

$$\frac{1}{\sqrt{G_0}} = 1 - \frac{\Delta m}{m} - \frac{\Delta m}{m}\frac{m_0 c^2}{T} \tag{3.6}$$

Substituting the experimental value, $|\Delta m/m| \sim 5 \times 10^{-4}$ with $T = 6$ GeV, into equation (3.6) and using the relation $G_0 = (1 + b)^2$, we obtain

$$|b| \sim 5 \times 10^{-4} \tag{3.7}$$

If our mass anisotropy is interpreted as a classical feature for the photon cloud which has been discussed to date as the self-energy of the particle in quantum electrodynamics, the following result is obtained.

Since the relation between the bare mass and the experimental mass (anisotropic mass) is given by equation (2.3.5)

$$m_* = \sqrt{(G_0)}M_0 = M_0 + bM_0 \tag{3.8}$$

the self-energy of the particle is estimated to be of the order of

$$\left|\frac{\delta m}{M_0}\right| = b \sim 5 \times 10^{-4} \tag{3.9}$$

where the self-energy is given by $\delta m c_0^2 = (m_* - M_0)c_0^2$. Although the value of self-energy obtained by the conventional quantum electrodynamics is divergent, one expects that a future theory might possibly make it small and finite.

4. *Concluding Remarks*

From a microscopic point of view, it is quite natural to consider the motion of the particle with anisotropic mass associated with a photon cloud rather than with isotropic mass, since the relativistic mass of the particle is not bare.

On the basis of the hypothesis that the particle mass is anisotropic rather than isotropic, we have investigated the relativistic motion of the particle. We find that all the physical laws derived in the present theory for the mass anisotropic particle have the same mathematical forms as those in the ordinary theory of special relativity. However, one should notice that our physical laws are expressed using anisotropic mass contrary to the ordinary case for which the corresponding laws are always expressed using isotropic mass. In this work, experimental mass clearly distinguished from bare mass in a legitimate way.

When the experimental mass of the particle is written as the sum of the bare mass and the self-energy

$$m_* = M_0 + \delta m$$

the self-energy is found to be of the order of $0{\cdot}0005 M_0$ on the basis of the experiment of electron acceleration. Although the value of self-energy calculated by conventional quantum electrodynamics is unfortunately infinite, we hope that a future theory might produce a finite value for the electron self-energy.

Finally, we note that our space-time still follows the ordinary Minkowski's geometry because G_0 is a constant for all reference frames and that the time dilation and the length contraction for the case under consideration are identical with those for the ordinary case.

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